Least Square Q-Kirchhoff Migration: Implementation and Application

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SUMMARY

Absorption effects caused by the anelastic nature of the earth lead to the attenuation of amplitude and distortion of phase of seismic waves. The so-called quality factor "Q" accounting for this absorption effect has to be included for correct imaging. The two main challenges in Q compensation are the ill-posed nature of the problem and the complex absorption patterns along ray paths in real geological structures. To tackle the instability caused by the ill-posed nature of the problem as well as maintain the correct compensation for high dipping structures, we propose least squares Q-Kirchhoff migration (LSQPSDM) in which absorption is incorporated into the Kirchhoff modelling operator and Q compensation is achieved naturally via inversion with proper sparse constraints. With better illumination and Q compensation, fault imaging is naturally enhanced and migration artefacts reduced through the proposed LSQPSDM. In contrast to standard least squares Kirchhoff migration, in our approach the inversion is approximated by inverse Hessian filtering to give a cost-effective solution. The proposed LSQPSDM approach has been applied to synthetic data for validation and a field dataset from NWS Australia. Better fault imaging and SNR are obtained compared to conventional Q migration.

Key words: Absorption, Least Square Migration, Q Compensation, Hessian Filter

INTRODUCTION

Absorption effects caused by the anelastic nature of the earth leads to the attenuation of amplitude and distortion of phase of seismic waves. Conventional acoustic migration, formulated as the adjoint operator of forward modelling (Claerbout, 1992), cannot account for this effect due to the non-unitarity of the modelling operator in highly attenuative geologic environments. This may produce images with poor illumination, reduced resolution, and wrong placement of reflectors. The so-called quality factor "Q" accounting for this absorption effect has to be included for correct imaging. The two main challenges in Q compensation are the ill-posed nature of the problem and the complex absorption patterns along ray paths in real geological structures. Conventional 1D inverse Q-filtering fails to address these two challenges and are only applicable under limited circumstances. Xie et al. (2009) proposed Q pre-stack depth migration (QPSDM) which compensates for absorption during migration by fully honouring ray paths, however, the ill-posed nature is still not well-addressed in the approach. This may result in over-boosted noise and migration artefacts masking high dipping structures including faults. Moreover, the anti-alias implementation in Kirchhoff migration further reduces the compensation of high dipping structures. To tackle the instability caused by the ill-posed nature of the problem as well as maintain the correct compensation for high dipping structures, we propose least squares Q-Kirchhoff migration (LSQPSDM) in which absorption is incorporated into the Kirchhoff modelling operator and Q compensation is achieved naturally via inversion with proper sparse constraints. The regularization consisting of prediction filters from reference substacks and sparse constraint in image domain is built into our inversion process to reduce migration artefacts and improve both common image gathers and the stack image. With better illumination and Q compensation, fault imaging is naturally enhanced through the proposed least squares Q-Kirchhoff migration. In contrast to standard least squares Kirchhoff migration, in our approach the inversion is approximated by inverse Hessian filtering (Wang et al., 2016; Khalil et al., 2016) to give a cost-effective solution. The proposed LSQPSDM approach has been applied to synthetic data for validation and a field dataset from NWS Australia. Better fault imaging and SNR are obtained compared to conventional Q migration.

METHOD

In standard Least Square Migration (LSM), observed scattered data d is related to the reflectivity model or image m via the linear modelling operator LQ:

$$d = L_Q m \tag{1}$$

where LQ is the Kirchhhoff visco-acoustic modeling operator as follows:

$$L_Q(x_s, x_r) = \frac{-1}{8\pi^3} \int w(x_s, y, x_r) \exp(-i\omega T)\, D(x_s, y, x_r, \omega) m(y) i\omega dy\, d\omega$$

and $w(x_s, y, x_r)$ is the demigration weight, y the image location, T the travel time, and (x_s, x_r) are the shot and receiver coordinates subject to different parameterization. $D(x_s, y, x_r, \omega)$ is the dissipation function with the form

$$D(x_s, y, x_r, \omega) = \exp(-\frac{\omega T^*}{2})\exp(\frac{i\omega}{\pi} T^* \ln(\frac{\omega}{\omega_0}))$$

where T^* is the dissipation time obtained via ray tracing where $T^* = \int \frac{1}{v \cdot Q} ds$ and Q is the absorption quality factor. ω_0 is the reference frequency. Equation (1) normally is solved in least square sense (Nemeth et al., 1999),

$$m = \text{argmin}_{m_0} \left\| d - L_Q m_0 \right\|^2 \tag{2}$$

Problem (2) has an explicit solution

$$m = (L_Q^T L_Q)^{-1} L_Q^T d \tag{3}$$

However, solving the inversion problem (3) for real applications is computationally expensive even with iterative methods. Inspired by Guitton's work on approximating the Hessian matrix via matching filter (Guitton, 2004) we can reformulate the inversion problem as a standard deconvolution problem, making it much cheaper to solve. The same technique can be applied to Least Square Q-PSDM solution (3).

Denote m_1 the initial migration image and d_1 the initial de-migration data, we have

$$m_1 = L_Q^T d, \qquad d_1 = L_Q m_1$$

Therefore from the re-migration image m_2, it is possible to estimate the inverse of Hessian matrix because

$$m_2 = L_Q^T d_1 = L_Q^T L_Q m_1$$

Suppose a bank of non-stationary filters B solves the following optimization problem

$$\text{argmin}_B \|m_1 - Bm_2\|^2 \tag{4}$$

Then B is a good estimation of $(L_Q^T L_Q)^{-1}$, and an improved image or approximation to (3) is obtained by convolution Bm_1.

It is worth mentioning that because the solution space for B is large, some sparse constraints have to be imposed on (4) to obtain meaningful results, and this is especially important for the modeling operator with absorption effect. The regularization term is ignored in the formula for simplification of equations in the paper.

Synthetic Data RESULTS

We first test the proposed method on a synthetic dataset as shown in Figure 1. Figure 1a shows the Q model we used. Two strong Q anomalies (20-30, in red colour) are present at shallow part, together with a relatively weaker Q (100-200, in blue colour) in background. Two sets of synthetic data are generated with our wave equation modelling, one using acoustic modelling, and another using visco-acoustic modelling. Figure 1b is the acoustic migration result with acoustic modelling data as a reference. Figure 1c shows the acoustic migration result with visco-acoustic modelling data. It is clear that without proper Q-compensation, migration result is of lower resolution and events beneath Q anomalies are much weaker. Comparing to the reference, the result of LSQPSDM in Figure 1d shows correct compensation, and spectrum comparison in Figure 1e further justifies the correctness of our method.

Real Data RESULTS

We further test our method on a real dataset from NWS Australia. For this data, a background earth absorption Q factor of 150 has been estimated and verified by client. Migration results are shown in Figure 2. Figure 2a is the conventional migration section without considering Q at offset 2000m. This image lacks high frequencies and resolution is lost due to earth absorption. QPSDM and LSQPSDM are presented in Figures 2b and 2c. Both methods are able to compensate for high frequency loss and improve resolution, but we can also notice that QPSDM suffers from migration artefacts and fault imaging is masked by over-boosted noise. In contrast, LSQPSDM provides sharper fault imaging and migration artefacts are reduced. Because the fault plane is less well illuminated, least squares migration can better compensate and enhance fault imaging. Improvement in fault imaging and the reduction in migration artefacts are also clearly observed in the stack comparison as shown in Figures 2d, 2e and 2f. Figure 3 shows a comparison of common image gathers (CIGs) with a 45 degree angle mute applied. The CIGs produced by LSQPSDM have higher signal-to-noise ratio and events become more continuous compared to QPSDM gathers. Two examples are highlighted by the small blue arrow in figure 3. On top of CIGs is the amplitude curve at event around 2500m pointed by big blue arrow. It is observed that both QPSDM and LSQPSDM give similar amplitude curve across offset.

CONCLUSIONS

We present a robust and efficient implementation of least square Kirchhoff Q-migration, in which absorption is incorporated into the Kirchhoff modeling operator and inversion is approximated by Hessian filtering. This new method compensates for absorption effects by honoring raypath and reduces migration artifacts through a regularization constraint. Better fault imaging and less migration artefacts/noise are achieved with LSQPSDM.

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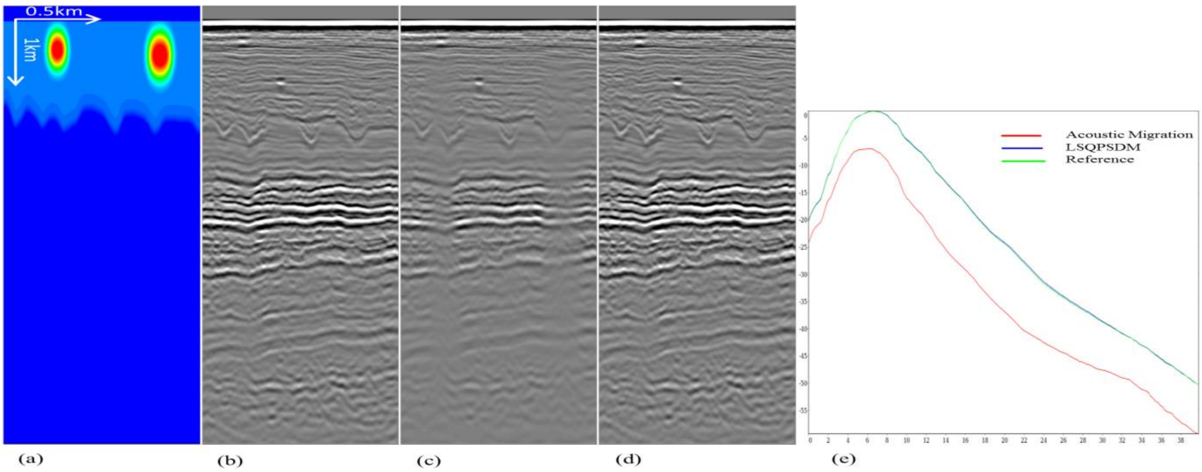

Figure 1 Comparison of Migration. (a) Q model, (b) Acoustic modelling and conventional acoustic PSDM, as a reference, (c) Visco-acoustic modelling and conventional acoustic migration, (d) Visco-acoustic modelling and LSQPSDM, (e) Spectrum Comparison.

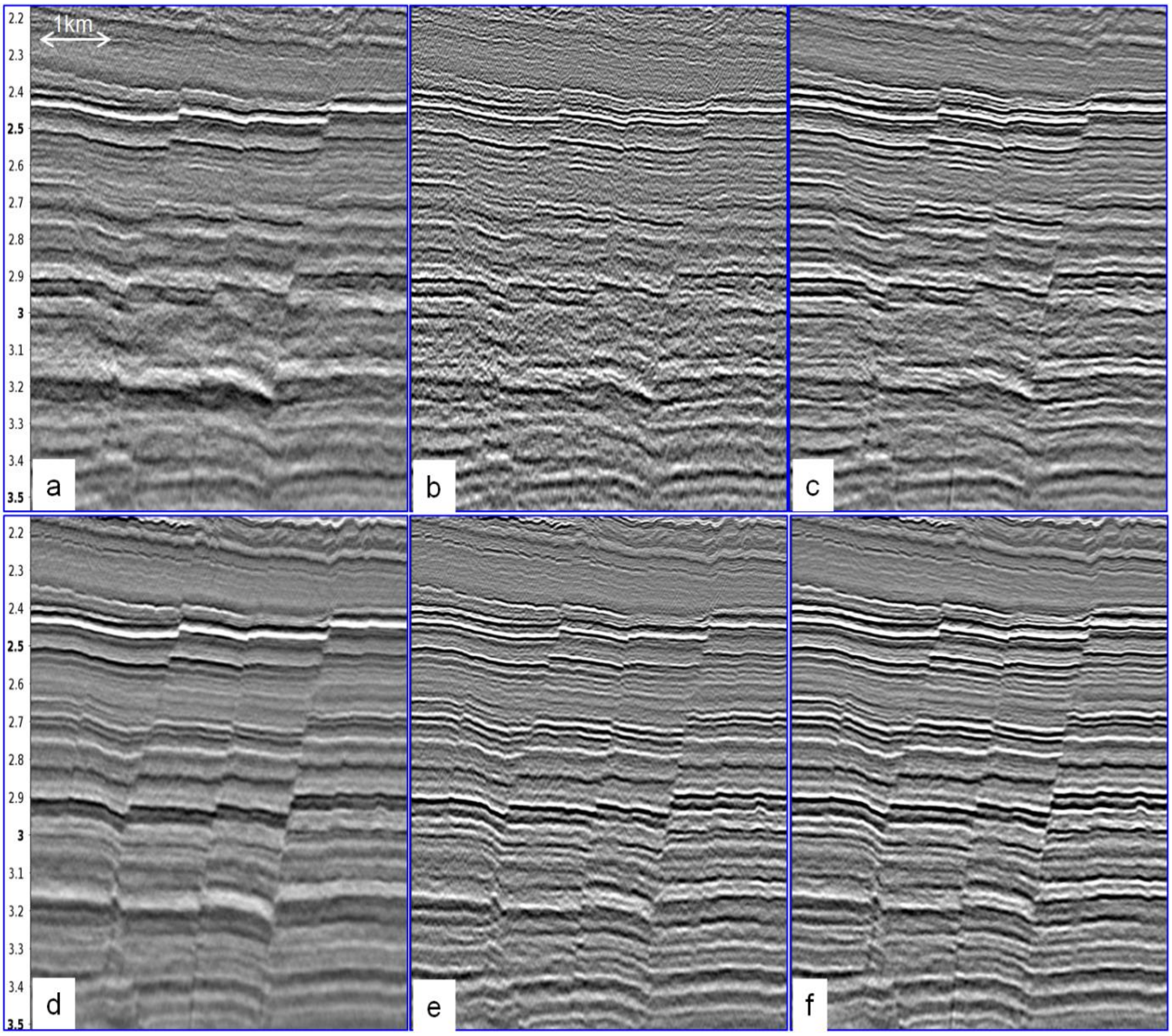

Figure 2 Comparison of migration sections. (a) Conventional migration at offset 2000m, (b) QPSDM at offset 2000m, (c) LSQPSDM at offset 2000m, (d) Conventional migration stack (e) QPSDM stack, (f) LSQPSDM stack.

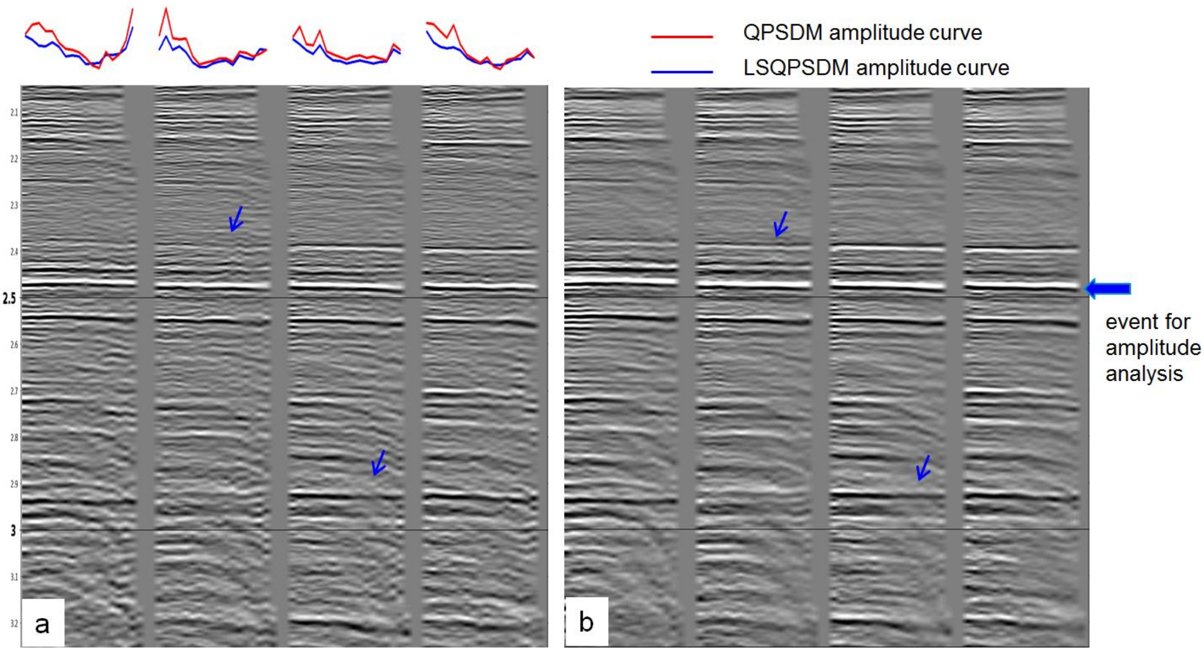

Figure 3 Comparison of CIGs. (a) PSDM (b) LSQPSDM.